

Fermilab theory seminar
July 19, 2007

QCD Factorization for Heavy Quarkonium Production at Collider Energies

Jianwei Qiu
Iowa State University/Argonne National Lab

Based on work with Z.-B. Kang, G.C. Nayak, and G. Sterman

Outline

- ❑ Why heavy quarkonium?
- ❑ Successes and difficulties in quarkonium production
- ❑ Connect pQCD factorization to NRQCD factorization
- ❑ Failure of conventional NRQCD factorization at NNLO
- ❑ Color transfer in associated production of heavy quarkonium
- ❑ Summary and conclusions

Why heavy quarkonium?

QWG Report, N. Brambilla et al, hep-ph/0412158

□ A good system for studying the confinement

Heavy quarkonium provides a non-relativistic system, potentially, very similar to a QED bound state:

Two intrinsic scales: large mass and small binding energy

$$\text{Charm: } \frac{v^2}{c^2} \sim \frac{k_Q^2}{m_Q^2} \sim \frac{|M^2 - 4m_c^2|}{4m_c^2} \sim 0.3 \qquad \text{Bottom: } \frac{v^2}{c^2} \sim 0.1$$

→ Heavy quark potential: $V_{Q\bar{Q}}(r)$

□ Offers a unique perspective to the hadronization

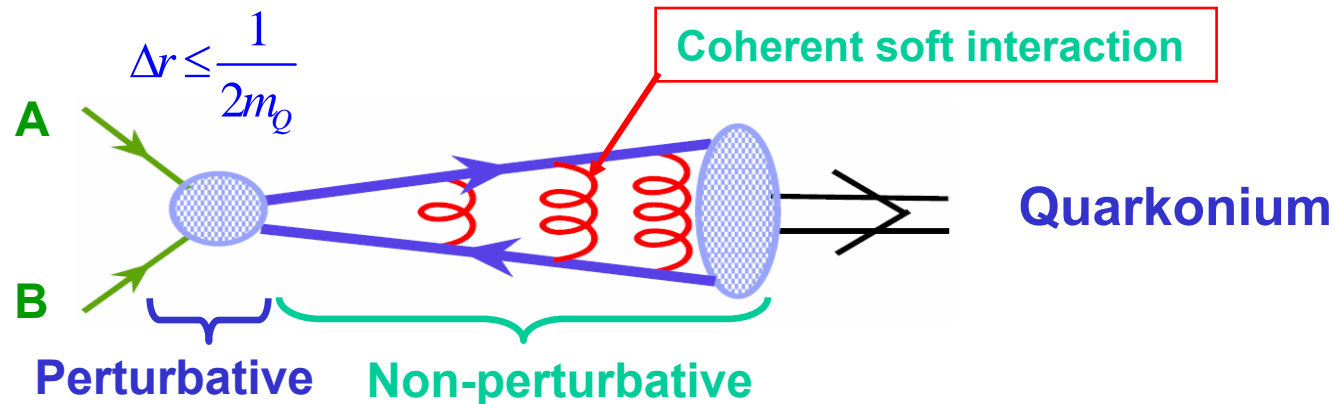
Production of heavy quarks is effectively perturbative:

Heavy quark pairs are produced at a distance scale much less than **fm**

$$\Delta r \sim \frac{1}{2m_Q} \leq 0.1 \text{ fm (for a charm-quark pair)} \\ \leq 0.025 \text{ fm (for a b-quark pair)}$$

The basic production mechanism

□ Production of a heavy quark pair:

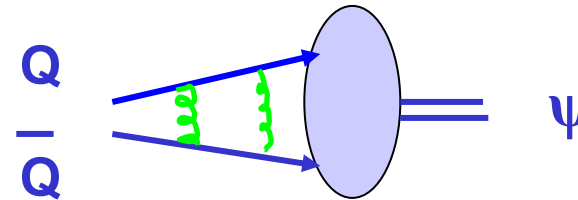
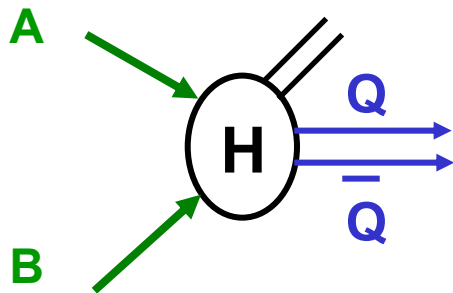


□ Hadronization of the pair → models:

$$\sigma_{AB \rightarrow h} = \sum_{\text{states}} \int d\Gamma_{Q\bar{Q}} \frac{d\sigma_{AB \rightarrow \text{states}(Q\bar{Q})}}{d\Gamma_{Q\bar{Q}}} F_{\text{states}(Q\bar{Q}) \rightarrow h} (p_Q, p_{\bar{Q}}, p_h)$$

Different models ⇔ Different assumptions/treatments on how the heavy quark pair becomes a quarkonium

Color singlet model



Einhorn and Ellis (1975), ...

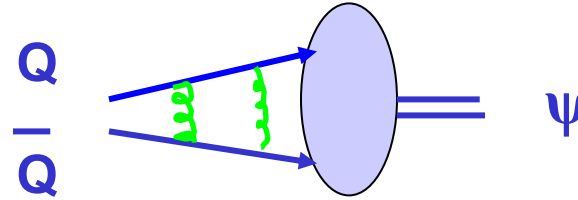
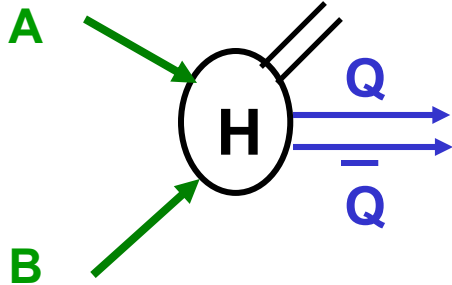
- ❖ color singlet pair
- ❖ right quantum numbers for the quarkonium
- ❖ same wave function for production and decay

- ❖ absolutely normalized predictions
- ❖ predictions on polarization
- ❖ quantum interference between production and formation suppressed

$$\sigma_{AB \rightarrow \psi} \propto \sigma_{AB \rightarrow (Q\bar{Q})} \left| R_{\psi}(0) \right|^2$$

Works well for J/ψ production in photo-production and others
But, one order of magnitude too small for CDF data, ...

Color Evaporation Model



Fritsch (1978); Halzen; ...

- ❖ all pairs with invariant mass less than open flavor threshold
- ❖ color and spin average
- ❖ a single constant for non-perturbative formation
- ❖ one constant for one quarkonium state

$$\sigma_{AB \rightarrow \psi} = f_{\psi} \int dm_{Q\bar{Q}}^2 \frac{d\sigma_{AB \rightarrow (Q\bar{Q})}}{dm_{Q\bar{Q}}^2}$$

**Works well for total cross sections, not perfect for distributions,
Predicts zero polarization for quarkonium production**

Non-relativistic QCD (NRQCD) model

Bodwin, Braaten, Lapage (1994); ...

- Work in the heavy quark pair's rest frame
- “Integrate out” heavy quark dynamics: $(\mathcal{O}(\alpha_s^n(m_Q)))$
- Factorize the hadronization: $(\mathcal{O}(v_{rel}^n))$
- IR divergences \longrightarrow universal local matrix elements

$$\sigma_{AB \rightarrow J/\psi} (M_{J/\psi}) \approx \sum_{[O]} \sigma_{AB \rightarrow [O]} (2m_{c\bar{c}} = M_{J/\psi}) \langle \mathcal{O}_{J/\psi}(0) \rangle$$

- Quantum states $[O]$ separated by **spin** and **color**
- Color octet *and* color singlet QQbar \longrightarrow quarkonium
- Approximations/assumptions – velocity expansion

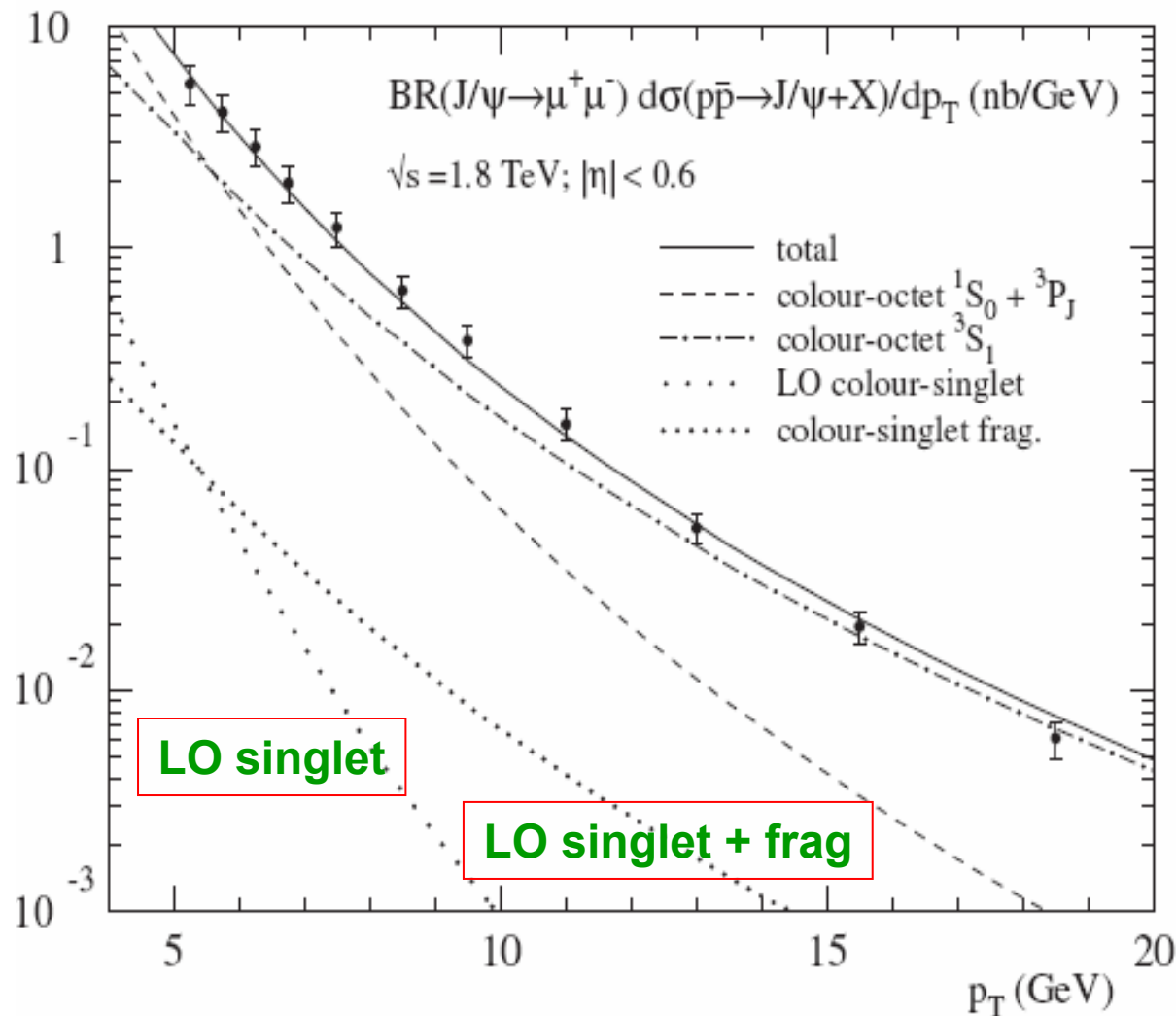
$$\langle p_Q - p_{\bar{Q}} \rangle \ll 2m_Q$$

It has been the most successful model

Successes of the production models

□ Unpolarized J/ψ at the Tevatron:

M. Kramer, 2001

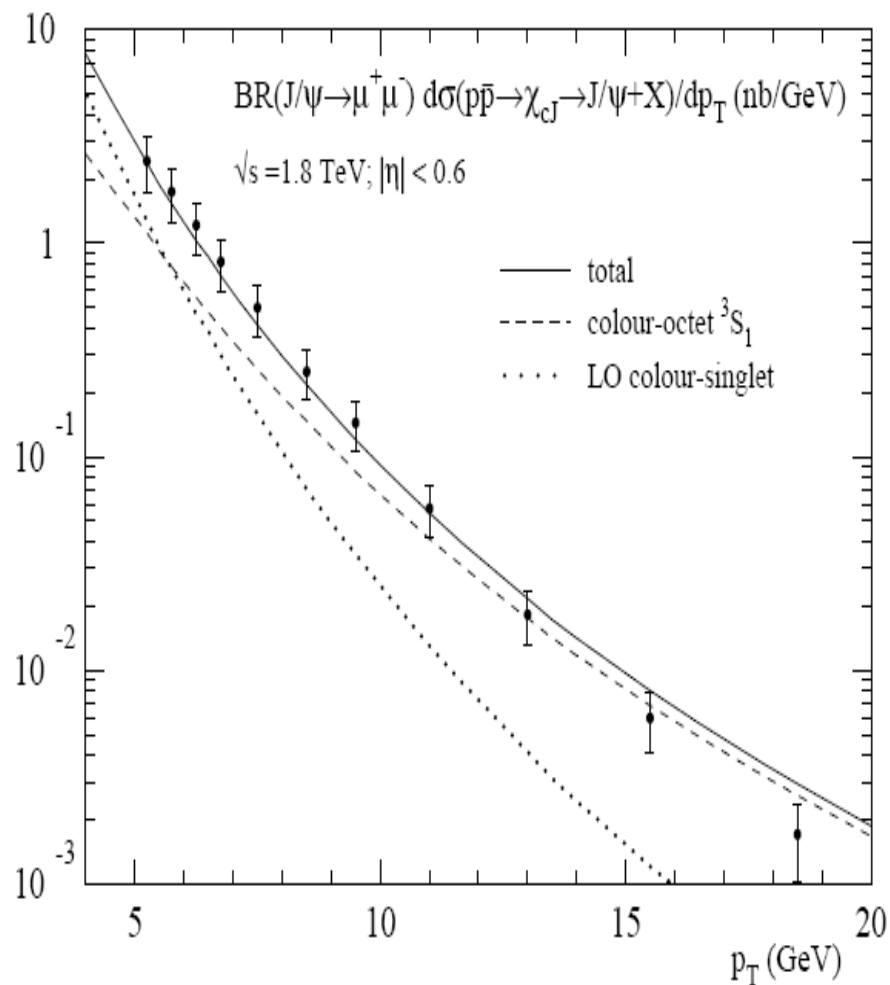
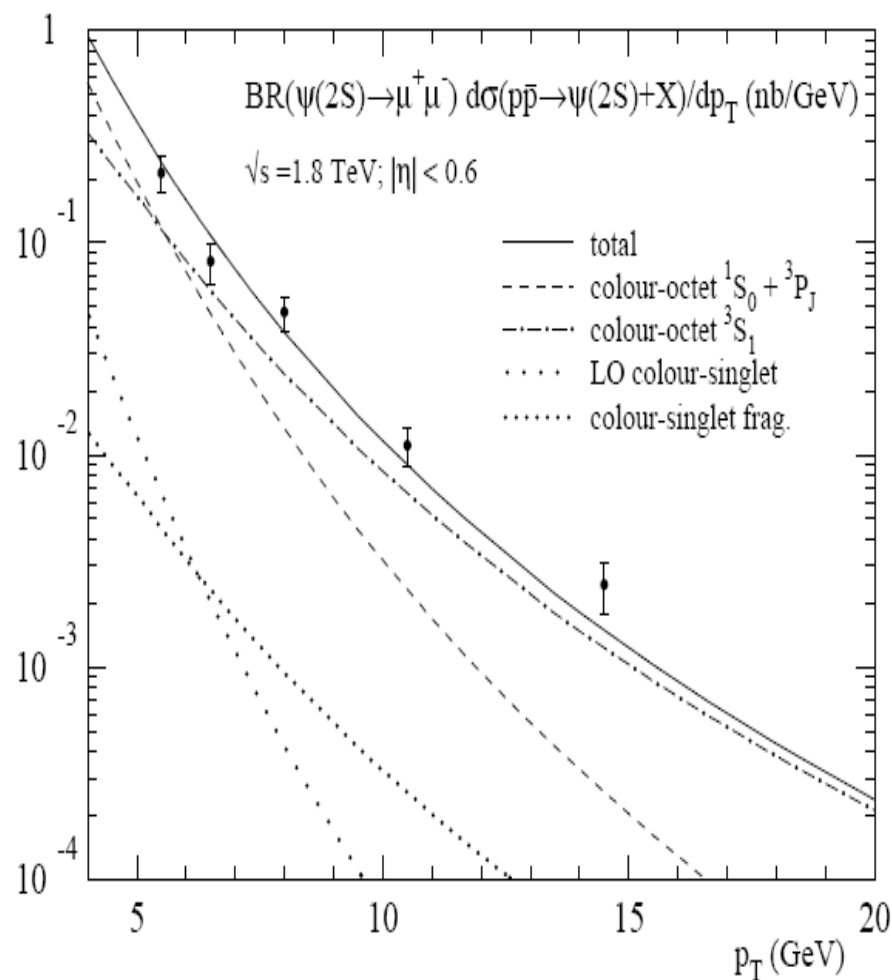


❖ Data is not consistent with color singlet model

❖ Data is consistent with NRQCD, with matrix elements fixed by the data

❖ CEM predicts a similar p_T distribution

□ Works for other states too:

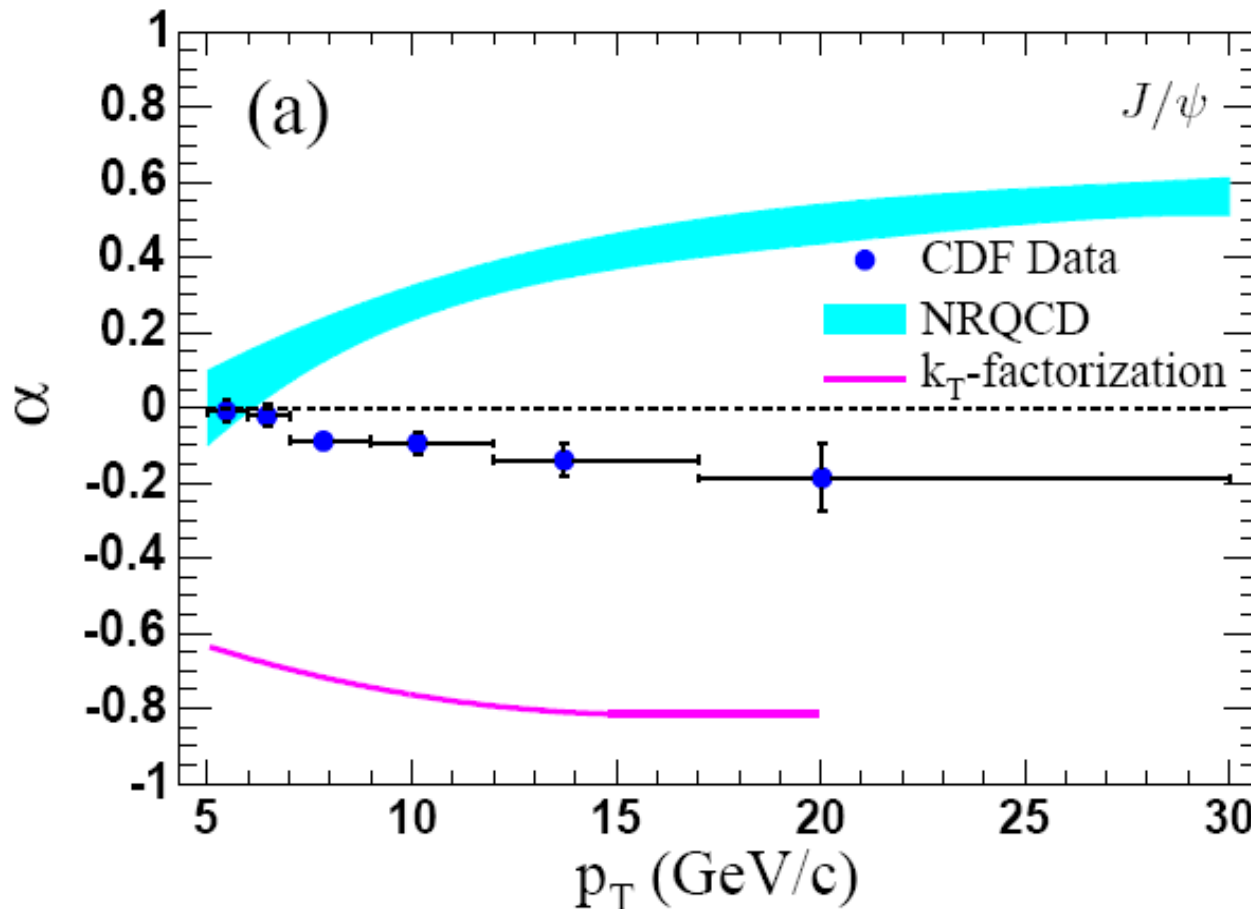


E. Braaten et al. Annu. Rev. Nucl. Part. Sci. 46, 197 (1996)

Difficulties

□ Transverse polarization at high p_T ?

NRQCD: Cho & Wise, Beneke & Rothstein, 1995, ...

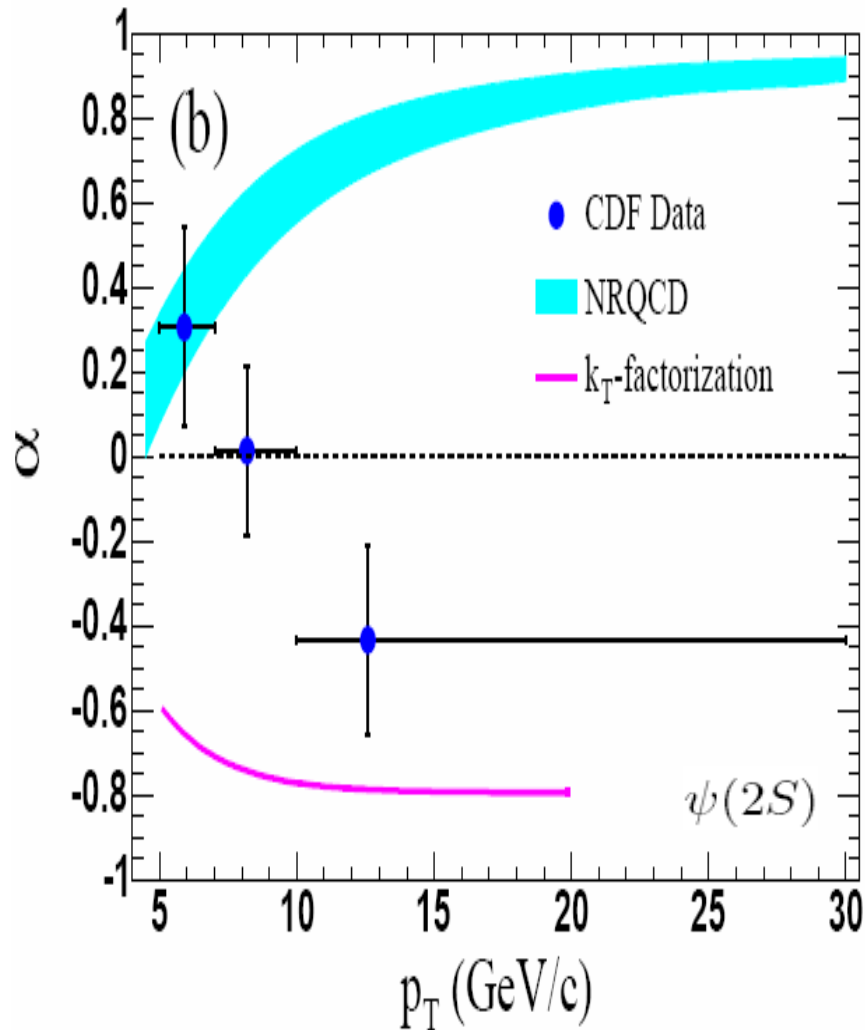


KT-fact: Baranov, 2002

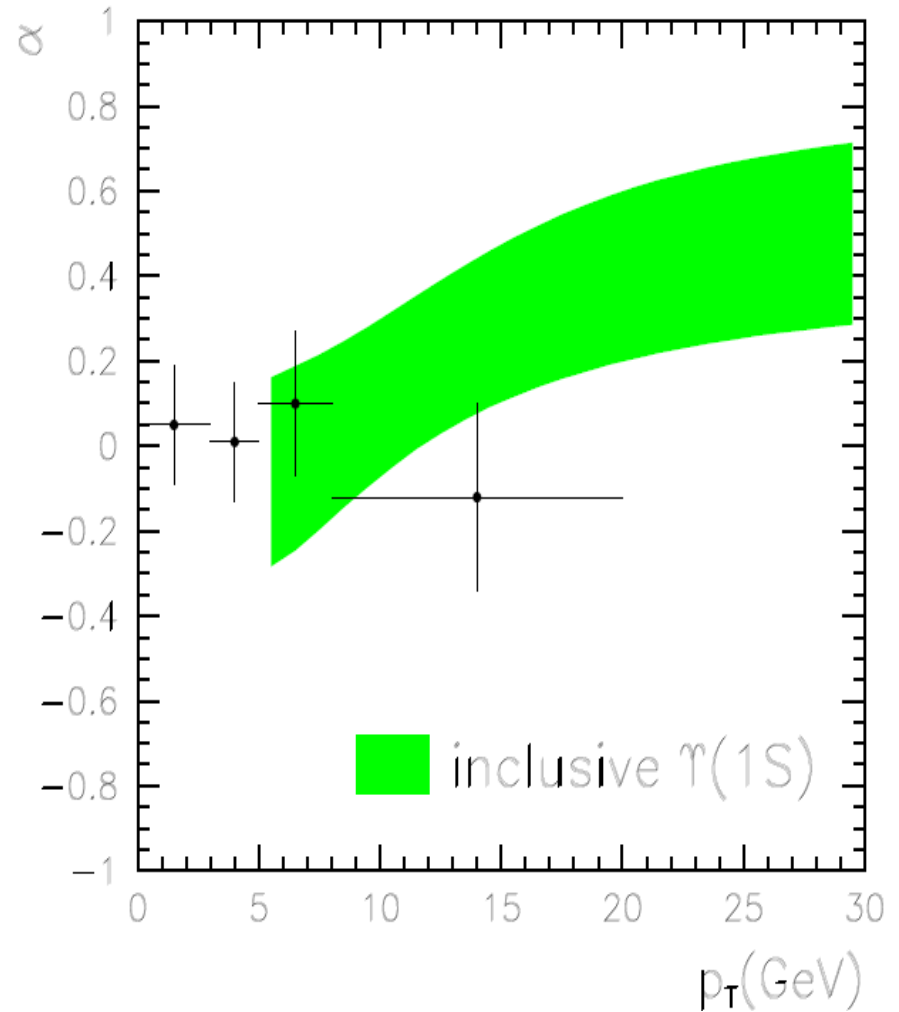
$$\alpha = \frac{\sigma_T - \sigma_L}{\sigma_T + \sigma_L}$$

CDF Collab. arXiv:0704.0638 [hep-ex]

❑ Same problem for other states:



CDF Collab. arXiv:0704.0638 [hep-ex]



Braaton & Lee, PRD63, 071501 (2001)

Double $c\bar{c}$ production in e^+e^-

□ Exclusive production:

[4] Li, He, and Chao, [6] Braaten and Lee

$J/\psi\ c\bar{c}$	$\eta_c(1S)$	χ_{c0}	$\eta_c(2S)$	
BABAR	$17.6 \pm 2.8^{+1.5}_{-2.1}$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	$16.4 \pm 3.7^{+2.4}_{-3.0}$	
Belle [14]	$25.6 \pm 2.8 \pm 3.4$	$6.4 \pm 1.7 \pm 1.0$	$16.5 \pm 3.0 \pm 2.4$	
NRQCD [6]	2.31 ± 1.09	2.28 ± 1.03	0.96 ± 0.45	} LO
NRQCD [4]	5.5	6.9	3.7	

□ Possible resolution for $J/\psi + \eta_c$:

❖ NLO correction: $K_{\text{factor}} = 1.96$

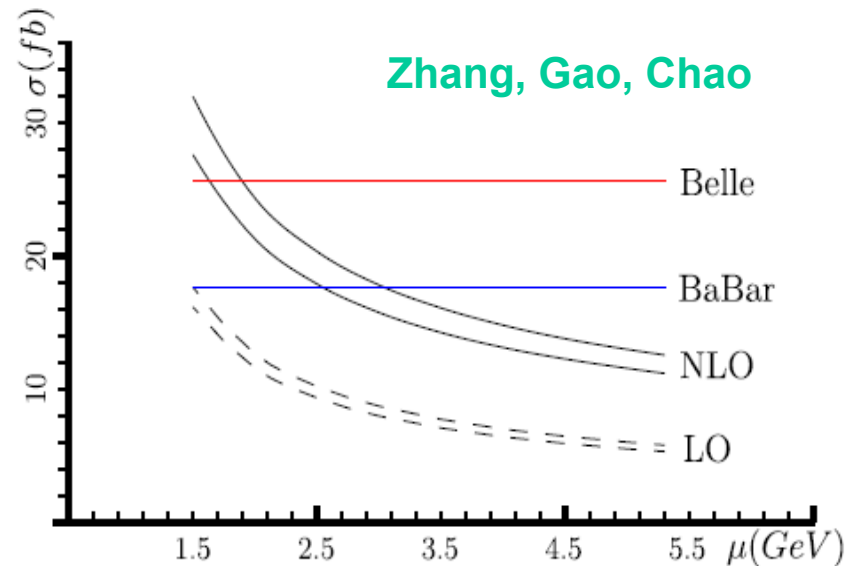
❖ Relativistic Correction:

X-section: $K_{\text{factor}} = 1.34$

Wave func: $K_{\text{factor}} = 1.32$

Combined: $K_{\text{factor}} = 4.15$

$$\sigma[e^+e^- \rightarrow J/\psi + \eta_c] = 17.5 \pm 5.7 \text{ fb}$$



Bodwin et al. hep-ph/0611002

Double $c\bar{c}$ production in e^+e^-

□ Inclusive production:

$$\sigma(e^+e^- \rightarrow J/\psi c\bar{c})$$

Belle: $(0.87_{-0.19}^{+0.21} \pm 0.17) \text{ pb}$

NRQCD: $\sim 0.07 \text{ pb}$

Kiselev, et al 1994,
Cho, Leibovich, 1996
Yuan, Qiao, Chao, 1997

□ Ratio to light flavors:

$$\sigma(e^+e^- \rightarrow J/\psi c\bar{c}) / \sigma(e^+e^- \rightarrow J/\psi X)$$

Belle: $0.59_{-0.13}^{+0.15} \pm 0.12$

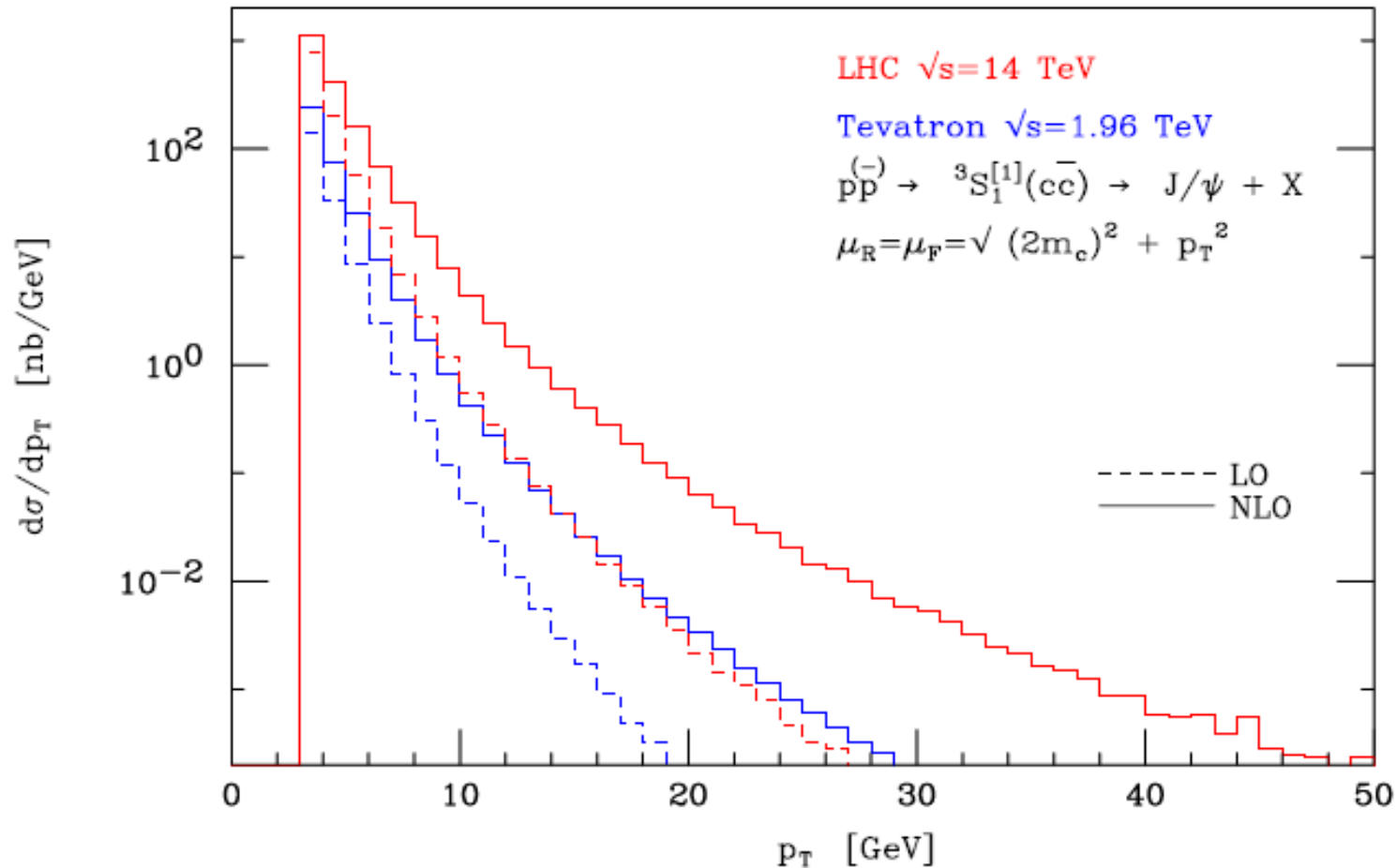
Message:

Production rate of $e^+e^- \rightarrow J/\psi c\bar{c}$ is larger than
all these channels: $e^+e^- \rightarrow J/\psi gg$, $e^+e^- \rightarrow J/\psi q\bar{q}$, ...
combined?

Large NLO correction

□ Color singlet at NLO:

Campbell, Maltoni, Tramontano, PRL98, 2007



Huge enhancement of NLO at high P_T

Questions

Is the NRQCD factorization valid for these observables?

- ☐ **NRQCD factorization is valid
for heavy quarkonium decay**


Bodwin, Braaten, Lepage, 1995

- ☐ **But, there is no rigorous proof for production
of heavy quarkonium**

Nayak, Qiu, Sterman, 2005/6

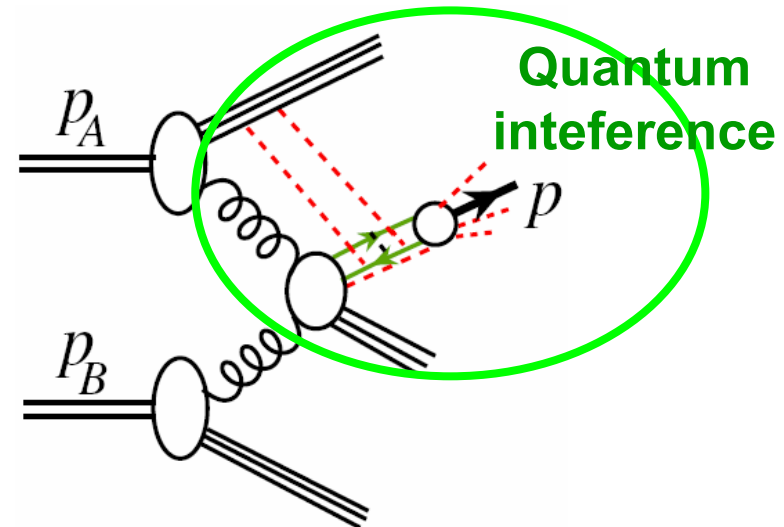
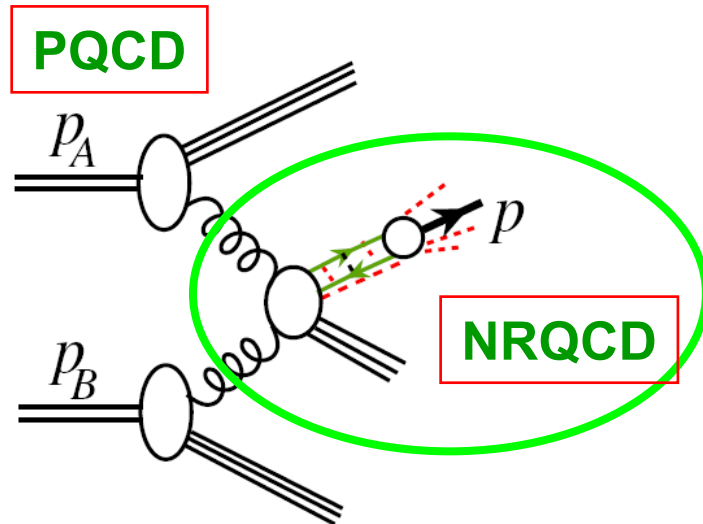
What we did on factorization

Nayak, Qiu, Sterman, 2005/6

- ❑ Study soft gluons in heavy quarkonium production at high p_T
- ❑ Find uncanceled infrared poles at NNLO not matched by conventional NRQCD matrix elements
- ❑ NNLO fix:
Gauge invariance  modification of NRQCD operators
- ❑ Get nonabelian phases: Wilson lines
- ❑ Factorization at high orders?
current state: can neither prove nor disprove

PQCD Factorization and Fragmentation

- ❑ None of the factorized production models, including NRQCD model, were proved theoretically
- ❑ Factorization of NRQCD model fails for **low p_T**

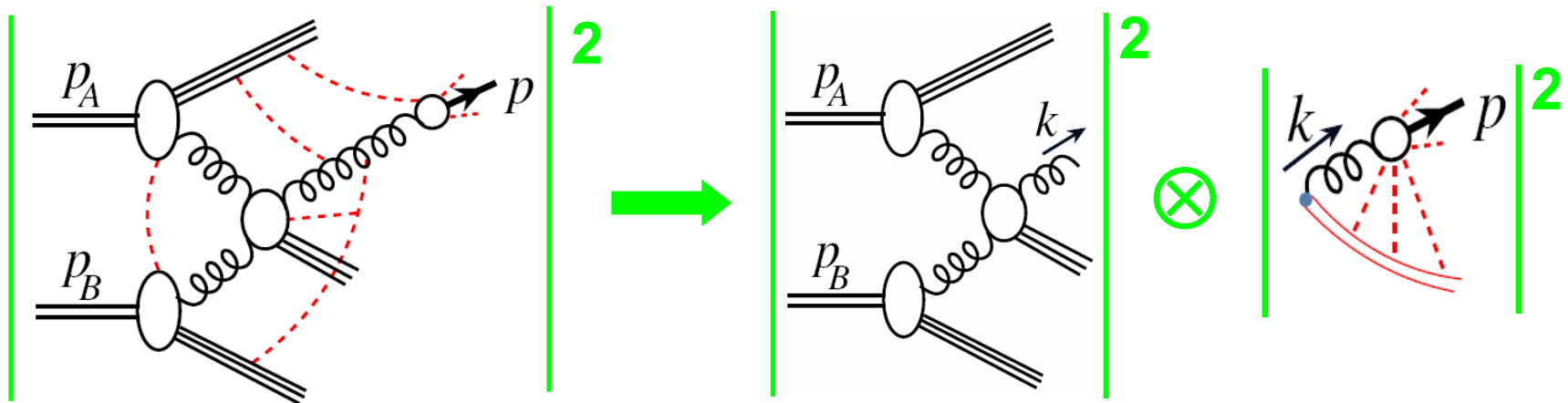


- ❑ Factorization of NRQCD model might work for **large p_T**
Spectator interactions are suppressed by $(1/p_T)^n$

Factorization is necessary for the predictive power

□ Factorization into a fragmentation function at **large p_T**

Nayak, Qiu, Stermen, 2005



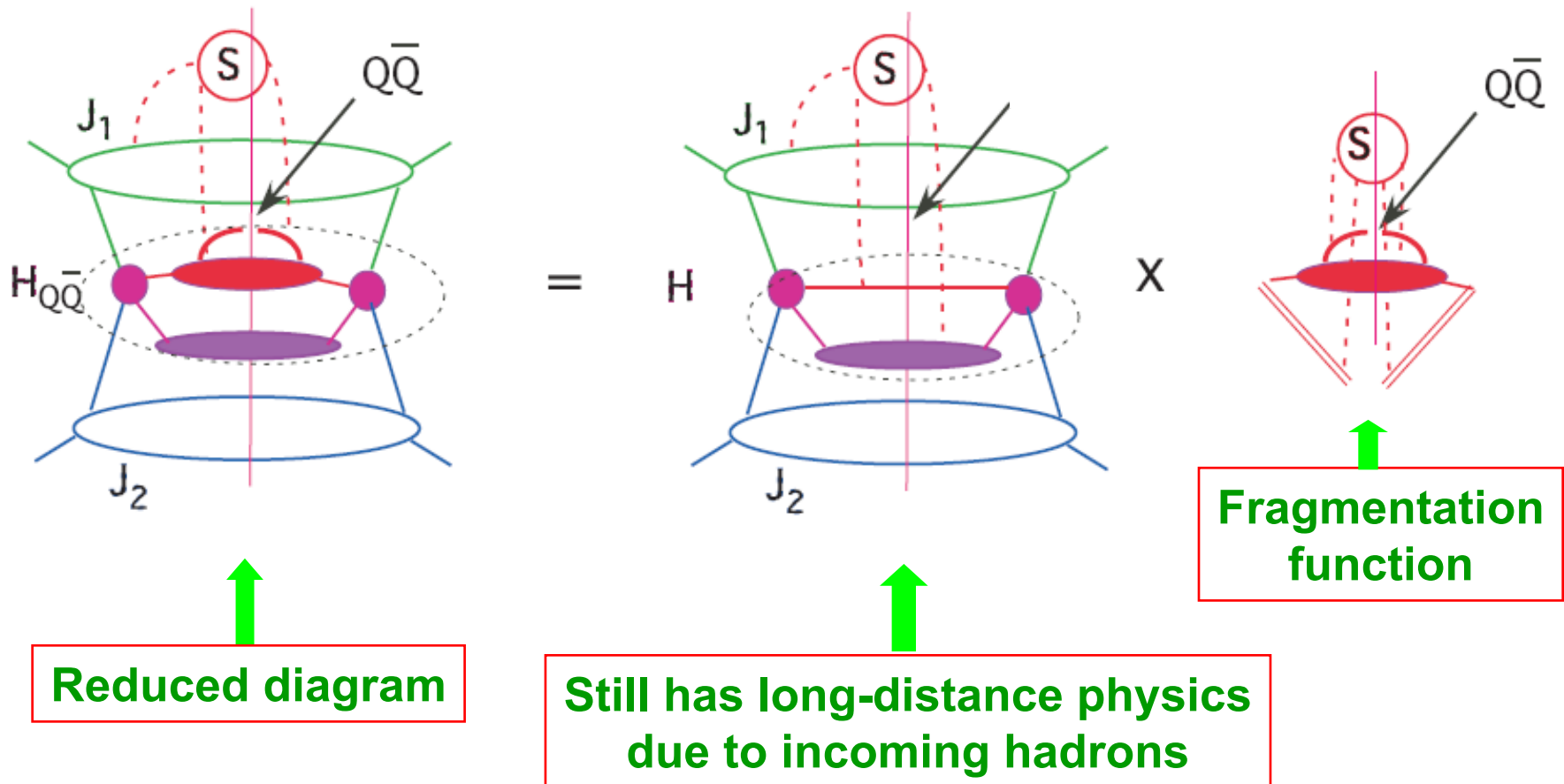
$$d\sigma_{A+B \rightarrow H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B \rightarrow i+X}(p_T/z, \mu) \otimes D_{H/i}(z, m_c, \mu) + \mathcal{O}(m_H^2/p_T^2)$$

□ Fragmentation function – gluon to a hadron H (e.g., J/ψ):

$$D_{H/g}(z, m_c, \mu) \propto \frac{1}{P^+} \text{Tr}_{color} \int dx^- e^{-ik^+ x^-} \\ \times \langle F^{+\lambda}(0) [\Phi_-^{(g)}(0)]^\dagger a_H(P^+) a_H^\dagger(P^+) \Phi_-^{(g)}(x^-) F_\lambda^+(y^-) \rangle$$

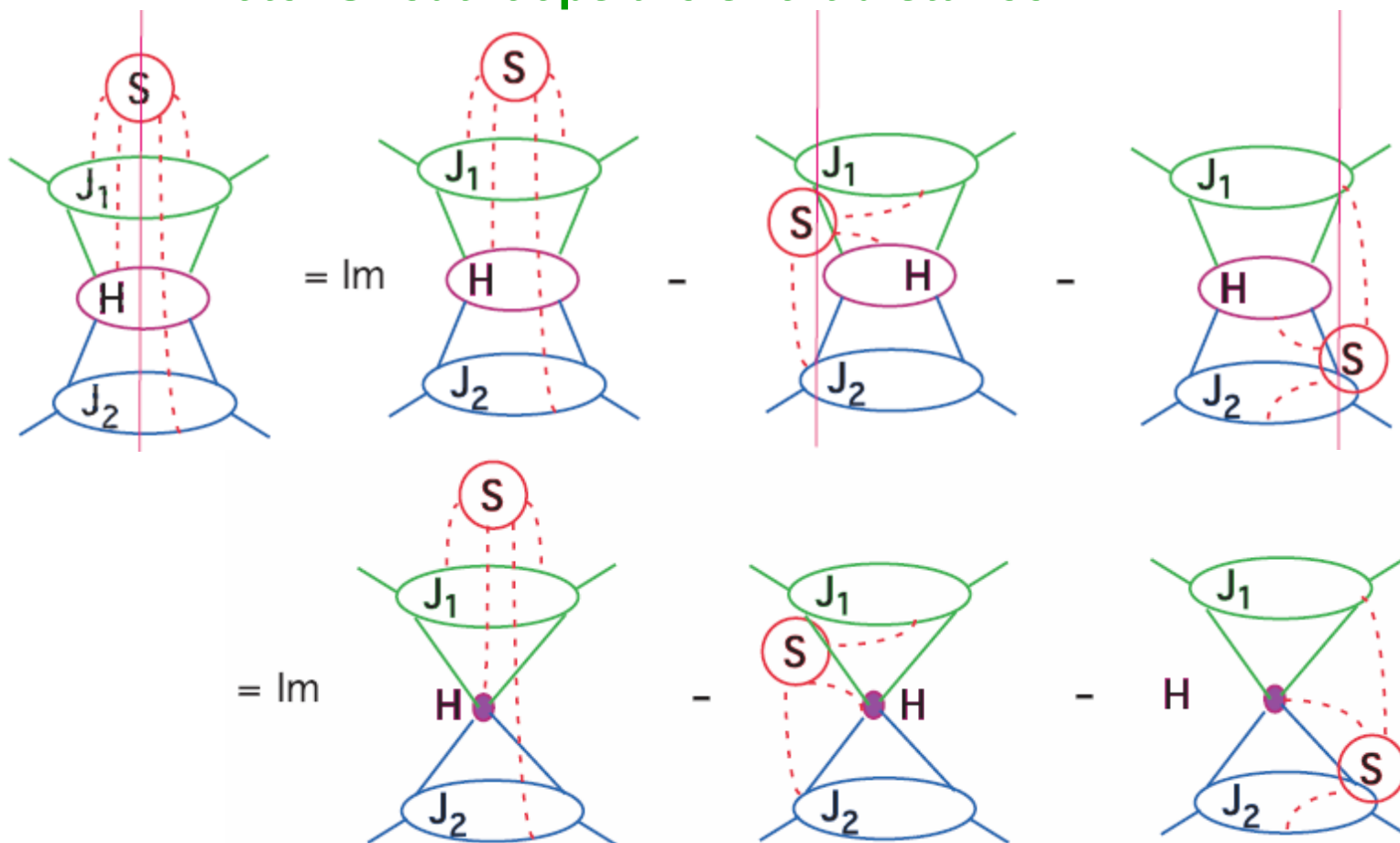
The proof works in two steps

□ Step 1: Fragmentation factorizes from the rest



□ Step 2: Cancellation of remaining IR final state:

Note: Uncut loops are short distance



Remaining soft-interaction absorbed into the Wilson lines of PDFs

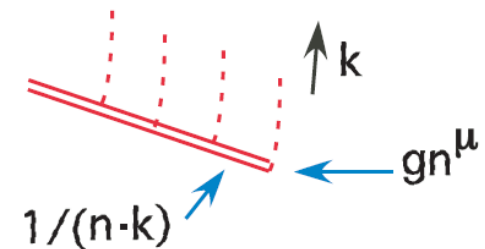
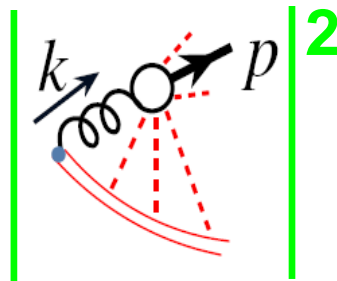
H is IR safe!

□ The Wilson line in x^- direction ($n^\mu = \delta_{\mu-}$)

$$\Phi_-^{(g)}(x^-) = P \exp \left[-ig \int_0^\infty n \cdot A^{(adj)}((x^- + \lambda)n) d\lambda \right]$$

Which depends on the “direction” vector: n^μ

□ For the fragmentation function, or the jet, all that is left is gluon source:



□ A necessary condition for the factorization, or the universality of the fragmentation function is:

The fragmentation function is independent of the n^μ

Connection to NRQCD Factorization


□ Proposed NRQCD factorization:

$$d\sigma_{A+B \rightarrow H+X}(p_T) = \sum_n d\hat{\sigma}_{A+B \rightarrow c\bar{c}[n]+X}(p_T) \langle \mathcal{O}_n^H \rangle$$

□ Proved pQCD factorization for single hadron production:

$$d\sigma_{A+B \rightarrow H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B \rightarrow i+X}(p_T/z, \mu) \otimes D_{H/i}(z, m_c, \mu) + \mathcal{O}(m_H^2/p_T^2)$$

□ Prove NRQCD Factorization

 **To prove:** $D_{H/i}(z, m_c, \mu) = \sum_n d_{i \rightarrow c\bar{c}[n]}(z, \mu, m_c) \langle \mathcal{O}_n^H \rangle$

with ❖ $d_{g \rightarrow c\bar{c}[n]}(z, \mu, m_c)$ **IR safe**

❖ $\langle \mathcal{O}_n^H \rangle$ **gauge invariant and universal**

❖ **independent of the direction of the Wilson lines**

Gauge Invariance and Wilson lines

- **Conventional operator definition** ($Q\bar{Q}$ rest frame)

$$\mathcal{O}_n^H(0) = \chi^\dagger \mathcal{K}_n \psi(0) \left(a_H^\dagger a_H \right) \psi^\dagger \mathcal{K}'_n \chi(0)$$

- ψ , χ are heavy quark, antiquark fields
- $\mathcal{K}_n, \mathcal{K}'_n$: **Products of color and spin matrices, covariant derivatives**
- **Fields at $x = 0$ but \mathcal{O}_n^H is not truly local**
- **Operator-valued gauge transformations**
(as to $A^+ = 0$ gauge) do not commute with $a_H^\dagger a_H$
- **Only color-singlet \mathcal{K} 's give gauge invariant \mathcal{O} 's**
or, the **color-octet** operators are **not** gauge invariant

□ Resolution: supplement fields by Wilson lines:

$$\Phi_l[x, A] = \exp \left[-ig \int_0^\infty d\lambda l \cdot A(x + \lambda l) \right]$$

□ Our new, gauge invariant operators:

$$\mathcal{O}_n^H(0) \rightarrow \chi^\dagger \mathcal{K}_{n,c} \psi(0) \Phi_l^\dagger[0, A]_{cb} \left(a_H^\dagger a_H \right) \Phi_l[0, A]_{ba} \chi^\dagger \mathcal{K}'_{n,a} \psi(0)$$

□ Two remaining questions for NRQCD factorization:

$$D_{H/i}(z, m_c, \mu) = \sum_n d_{i \rightarrow c\bar{c}[n]}(z, \mu, m_c) \langle \mathcal{O}_n^H \rangle$$

❖ Are the “coefficient” functions $d_{g \rightarrow c\bar{c}[n]}(z, \mu, m_c)$ IR safe?

Our NNLO answer is **no**  The lines are necessary

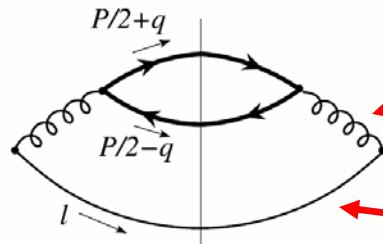
❖ Do the lines absorb all IR divergences?

Can't tell yet for sure. OK at NNLO

**An all-order proof of NRQCD factorization at high pT
is still lacking, and urgently needed**

Factorization works to NLO at v^2

□ LO:



Short-distance

Eikonal line

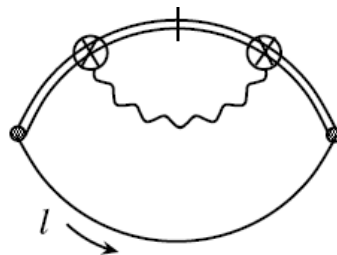
$n=[Q\bar{Q}]_{\text{singlet}}$

Velocity expansion:

$$\text{Diagram 1} + \text{Diagram 2} = \text{Diagram 3} + O(v^2)$$

The diagram shows the velocity expansion of a quark-gluon vertex. The first two diagrams are summed to equal the third diagram plus higher-order terms $O(v^2)$.

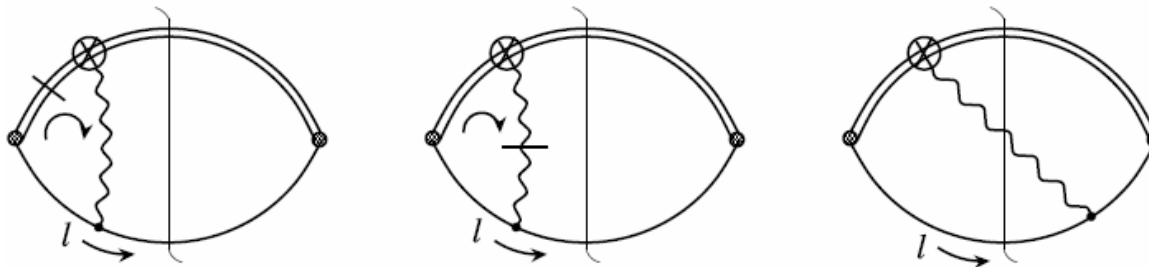
□ NLO:



$$= \frac{16}{3} \frac{\alpha_s}{\pi} \frac{\vec{q}^2}{P^2} \frac{1}{-\varepsilon} + \dots$$

Topologically-factorized
the matrix element $\langle \mathcal{O}_n^H \rangle$

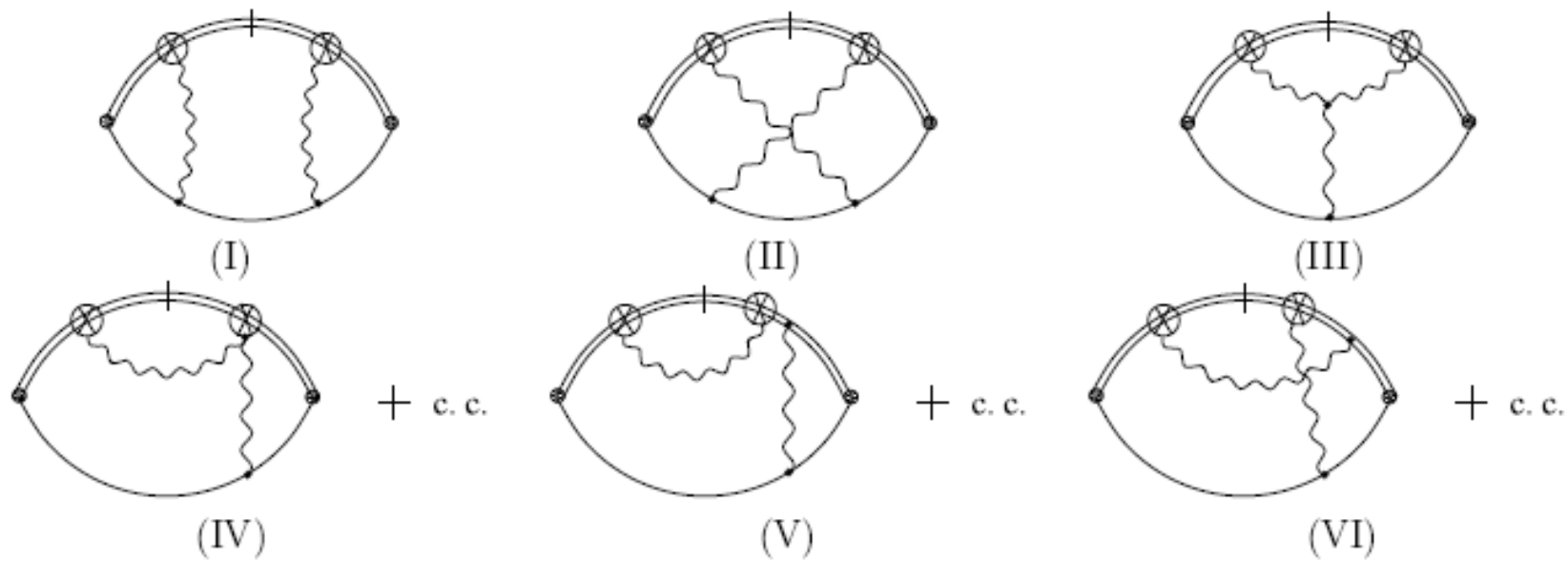
Color neutralization is IR divergent – nonperturbative!



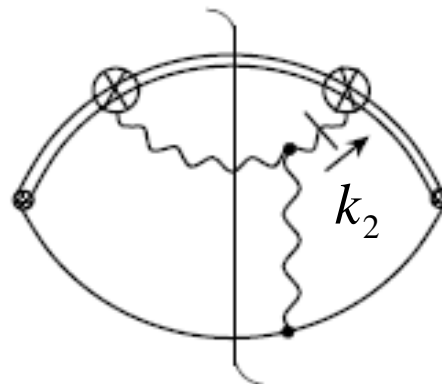
IR divergences cancel between real and virtual diagrams

Factorization fails at NNLO

□ Diagrams:



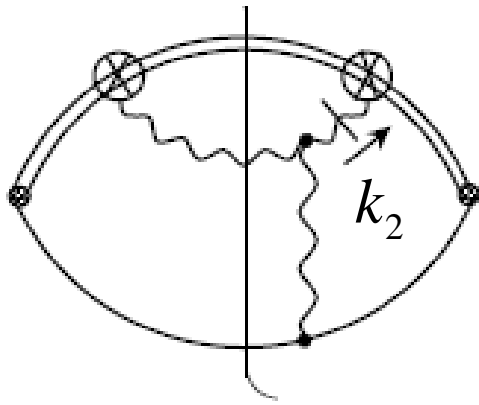
□ All IR divergences cancel between real and virtual diagrams, except



Explicit calculation at NNLO at v^2 – I

□ The infrared divergent expression to order $q^2 \sim v^2$:

$$\begin{aligned} \Sigma^{(2c)}(P, q, l) &= -16i g^4 \mu^{4\epsilon} \int \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_2}{(2\pi)^D}, 2\pi \delta(k_1^2) l^\lambda V_{\nu\mu\lambda}[k_1, k_2] \\ &\times [q^\mu (P \cdot k_1) - (q \cdot k_1) P^\mu] [q^\nu (P \cdot k_1) - (q \cdot k_2) P^\nu] \\ &\times \frac{1}{[P \cdot k_1 + i\epsilon]^2 [P \cdot k_2 - i\epsilon]^2} \\ &\times \frac{1}{[k_2^2 - i\epsilon] [(k_2 - k_1)^2 - i\epsilon] [l \cdot (k_1 - k_2) - i\epsilon]}, \end{aligned}$$



□ The result is:

$$\Sigma^{(2)}(P, q, l) = \alpha_s^2 \frac{4}{3\epsilon} \left[\frac{(P \cdot q)^2}{P^4} - \frac{q^2}{P^2} \right]$$

Explicit calculation at NNLO at v^2 - II

□ In heavy quark pair's rest frame:

$$\Sigma(P, q, l) = \alpha_s^2 \frac{4}{3\varepsilon} \frac{\vec{q}^2}{4m_c^2} = \alpha_s^2 \frac{1}{3\varepsilon} \frac{\vec{v}^2}{4}$$

□ IR poles would appear in coefficient function at NNLO
unless we have eikonal interactions to absorb them

□ This non-topological IR divergence Cannot be
absorbed into the conventional NRQCD matrix elements

□ Can be absorbed into the modified matrix elements

Factorization at a finite v ?

□ Velocity expansion is not efficient for charmonium

- ❖ Large phase space available for gluon radiation:

$$Q^2 - 4M_c^2 \Rightarrow 4M_D^2 - 4M_c^2 \approx 6 \text{ GeV}^2$$

- ❖ Large possible velocity in production:

$$v_{\text{prod}} \sim \frac{|k_c|}{M_c} \sim \sqrt{\frac{4M_D^2 - 4M_c^2}{4M_c^2}} \sim 0.88$$

- ❖ Very different from decay:

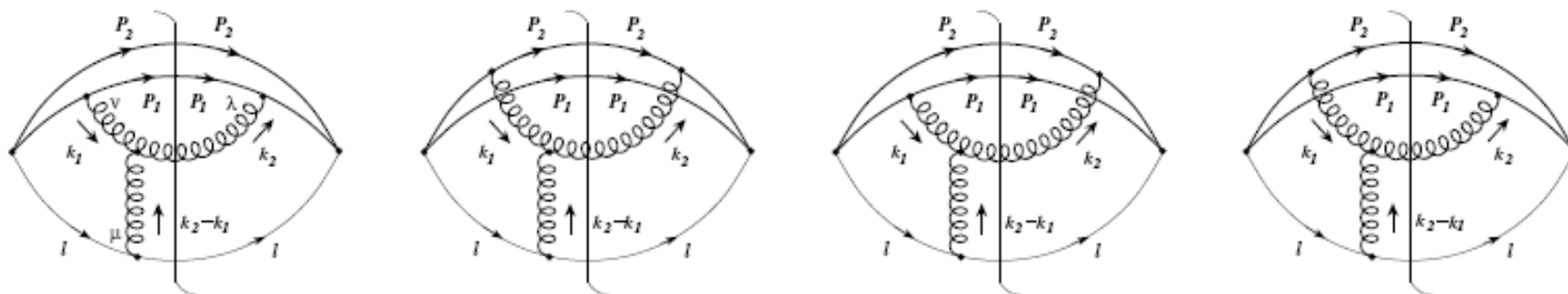
$$v_{\text{decay}} \sim \sqrt{\frac{4M_{J/\psi}^2 - 4M_c^2}{4M_c^2}} \sim 0.48$$

□ High order terms are very important

Still no solution for the polarization data
even if NRQCD factorization is valid

Factorization at NNLO and all order in v^2

□ Calculation with a finite v



$$\mathcal{I}^{(8 \rightarrow 1)} = \frac{\alpha_s^2}{4\epsilon} \left[1 - \frac{1}{2f(|\vec{v}|)} \ln \left[\frac{1 + f(|\vec{v}|)}{1 - f(|\vec{v}|)} \right] \right]$$

with

$$f(v) = \frac{2v}{1 + v^2} \quad \vec{v} = \vec{q}/E^*$$

$2E^*$ is the total energy of the heavy quark pair
($Q\bar{Q}$ rest frame)

□ Reproduce the v^2 result when expanded

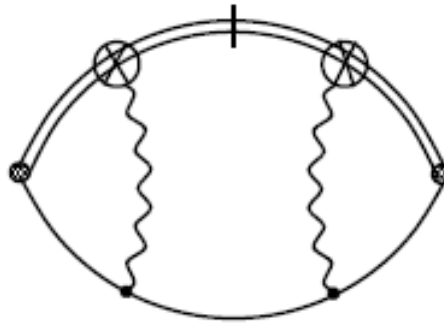
Significance?

- ❑ The IR poles at all orders of v -expansion at NNLO are **independent** of the direction of the Wilson line and **universal** - consistent with factorization
- ❑ Although limited to NNLO, our result suggests that the decoupling of **light parton** dynamics from heavy quark pair production is robust in perturbation theory at the level of infrared divergence - high orders?
- ❑ Although the eikonal approximation do not cover many terms in general NRQCD velocity expansion, in particular, those dealing with spin, it should cover all perturbative infrared divergences

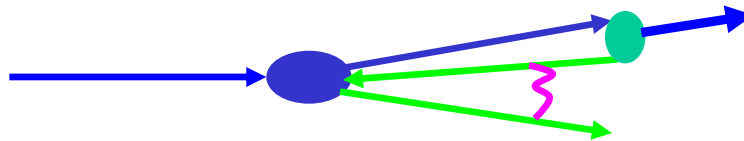
Associate production

□ Observation from NNLO calculation:

Double IR poles may not be canceled for a massive eikonal line



□ Color transfer in associated heavy quarkonium production:



A heavy quark as a color source to enhance the rate for an octet pair to become a singlet pair

Inclusive double $c\bar{c}$ production in e^+e^-

□ **Recall:** $\sigma(e^+e^- \rightarrow J/\psi c\bar{c})$

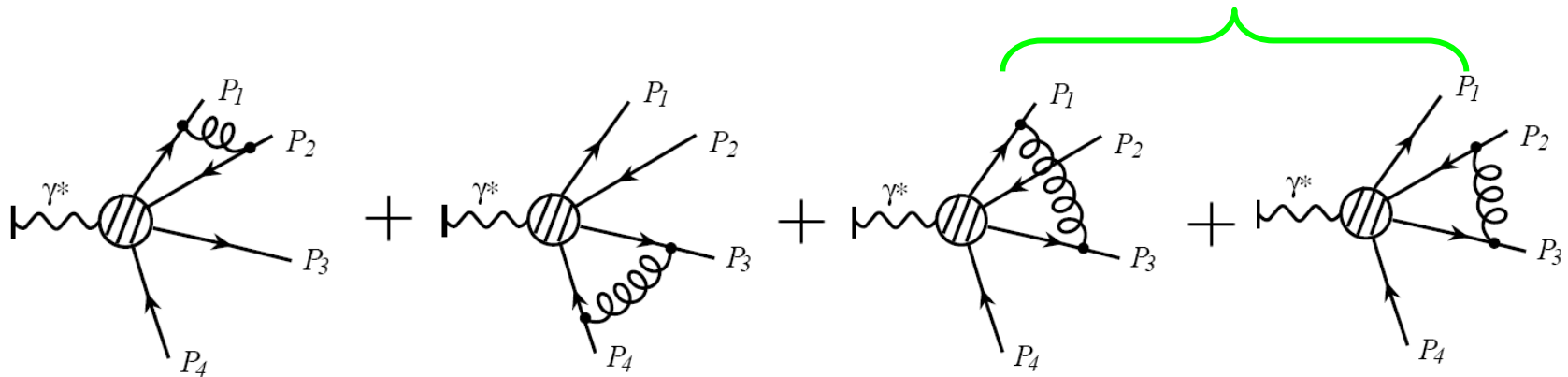
Belle: $(0.87^{+0.21}_{-0.19} \pm 0.17) \text{ pb}$

NRQCD: $\sim 0.07 \text{ pb}$

$$\sigma(e^+e^- \rightarrow J/\psi c\bar{c}) / \sigma(e^+e^- \rightarrow J/\psi X)$$

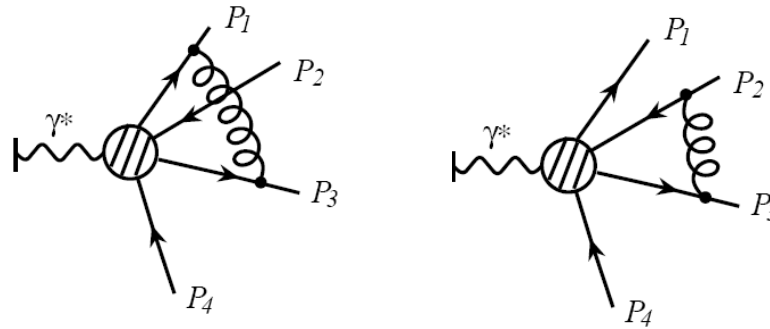
Belle: $0.59^{+0.15}_{-0.13} \pm 0.12$

□ **Associated production is enhanced:**



Soft gluon between heavy quarks

- Active pair: P_1, P_2
Spectators: P_3, P_4



- NRQCD does not work
when 3 heavy (anti-)quarks are close together:

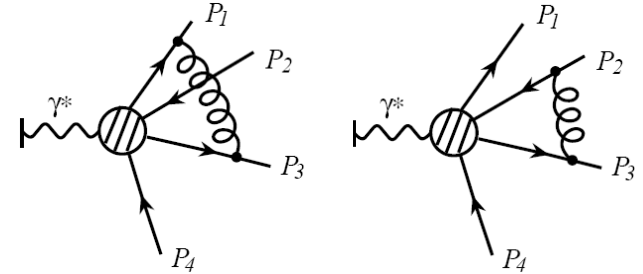
There are now 3 “velocities”: $\beta_{ij} \equiv \sqrt{1 - 4m^2/(P_i + P_j)^2}$

- Soft gluon:

$$\begin{aligned}
 & -i \int \frac{d^D k}{(2\pi)^D} \frac{4P_i \cdot P_j}{[2P_i \cdot k + k^2 + i\epsilon][-2P_j \cdot k + k^2 + i\epsilon][k^2 + i\epsilon]} \\
 & = \frac{\alpha_s}{2\pi} \left[-\frac{1}{2\epsilon} \left(\frac{1}{\beta_{ij}} + \beta_{ij} \right) (2\beta_{ij} - i\pi) + \dots \right] \Rightarrow i \frac{1}{\epsilon} \frac{\pi}{\beta_{ij}}
 \end{aligned}$$

NNLO enhancement

□ NLO correction to the amplitude:



$$\text{Im} [\mathcal{A}_{13} + \mathcal{A}_{23}] = \frac{\alpha_s}{4\varepsilon} \mathcal{A}^{(0)}(P_i) \left[\frac{1 + \beta_{13}^2}{\beta_{13}} - \frac{1 + \beta_{23}^2}{\beta_{23}} \right]$$

□ Expansion of relative velocity:

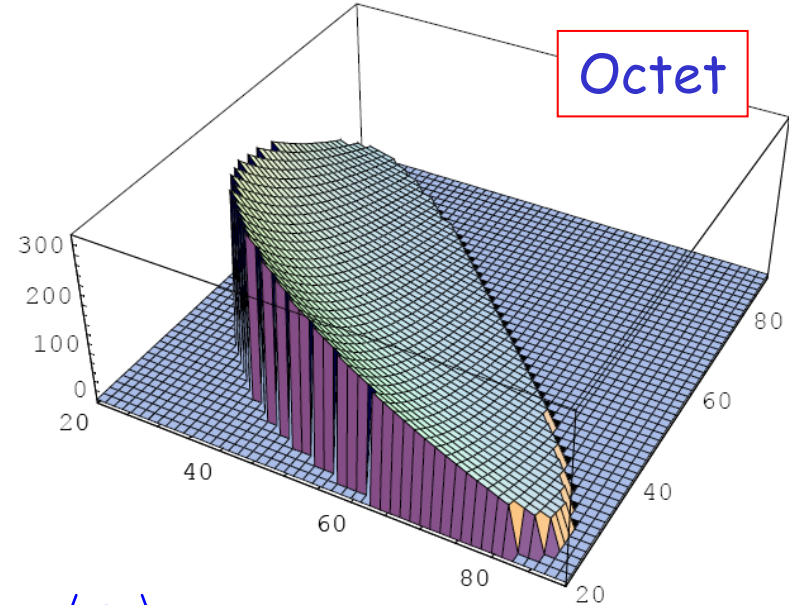
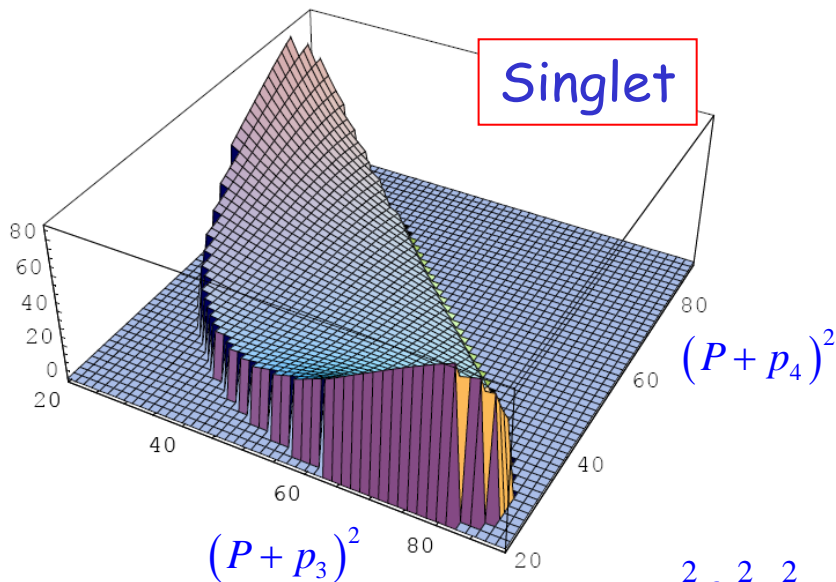
$$\frac{1}{\beta_{13}} - \frac{1}{\beta_{23}} \sim -\frac{2}{\beta_S^3} \frac{q_S \cdot q}{m^2} \sim \frac{2}{\beta_S^2} v \cos \phi_S \quad \begin{aligned} 2q_S &= P_3 - (P_1 + P_2)/2 \\ \beta_S &\sim \sqrt{-q_S^2}/m \end{aligned}$$

□ Enhancement factor from NNLO:

$$\left| A_{\text{Singlet}}^{\text{NNLO}} \right|^2 \sim \left(C_{8 \rightarrow 1} \frac{\alpha_s^2 v^2}{\varepsilon^2} \right) \left(\frac{\pi^2}{\beta_s^4} \right) \left| A_{\text{Octet}}^{\text{LO}} \right|^2$$

Estimate for the NNLO enhancement

□ LO hard parts with color factor:



□ Matrix elements:

$$\frac{\pi^2 \alpha_s^2 v^2}{\varepsilon^2} \Rightarrow \langle O_8 \rangle$$

$$d\sigma_{e^+e^- \rightarrow H+X}^{\text{tot}}(p_H) \sim d\hat{\sigma}_{e^+e^- \rightarrow Q\bar{Q}[S_1]+Q'(\beta_S)}(p_H) \langle {}^3\mathcal{S}_1^H \rangle \\ + d\hat{\sigma}_{e^+e^- \rightarrow Q\bar{Q}[S_8]+Q'(\beta_S)}(p_H) \frac{\langle {}^3\mathcal{S}_8^H \rangle}{\beta_S^4}$$

Two terms are equally important if $\beta_s \sim 0.3$

Same feature for heavy quark fragmentation, enhances long. Pol.

Summary and conclusions

- Predictive power of pQCD calculation relies on the factorization:
 - ❖ Infrared Safety of the short-distance part
 - ❖ Universality of the parton distribution/fragmentation
- None of the existing factorized production models, including NRQCD model, were proved theoretically
- We show that “NRQCD”-type factorization is valid up to the NNLO order in α_s for the fragmentation function
- Effective velocity in quarkonium production could be much larger than that in quarkonium decay

Summary and conclusions

- ❑ Associated production of heavy quarkonium could be strongly enhanced due to soft color transfer
- ❑ NRQCD does not work for associated production due to multiple “velocities”
- ❑ After more than 30 years, since the discovery of J/ψ , we still have not been able to fully understand the production mechanism of heavy quarkonia
- ❑ A tough question, but should have an interesting answer

Thank you!

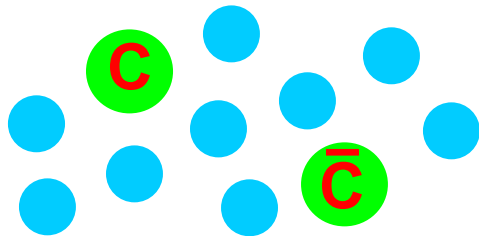
Backup slices

□ Could be a good probe for Quark Gluon Plasma (QGP)

- ❖ The transition from a heavy quark pair to a quarkonium should be sensitive to the soft physics or medium properties

Quarkonium binding energy: $\frac{|M^2 - 4m_Q^2|}{4m_Q^2} \ll 1$

- ❖ Color screening in QGP suppresses the formation of J/ψ



Potential: $V_{Q\bar{Q}}(r) \Rightarrow V_{Q\bar{Q}}(r, T)$ Matsui & Satz (1986)

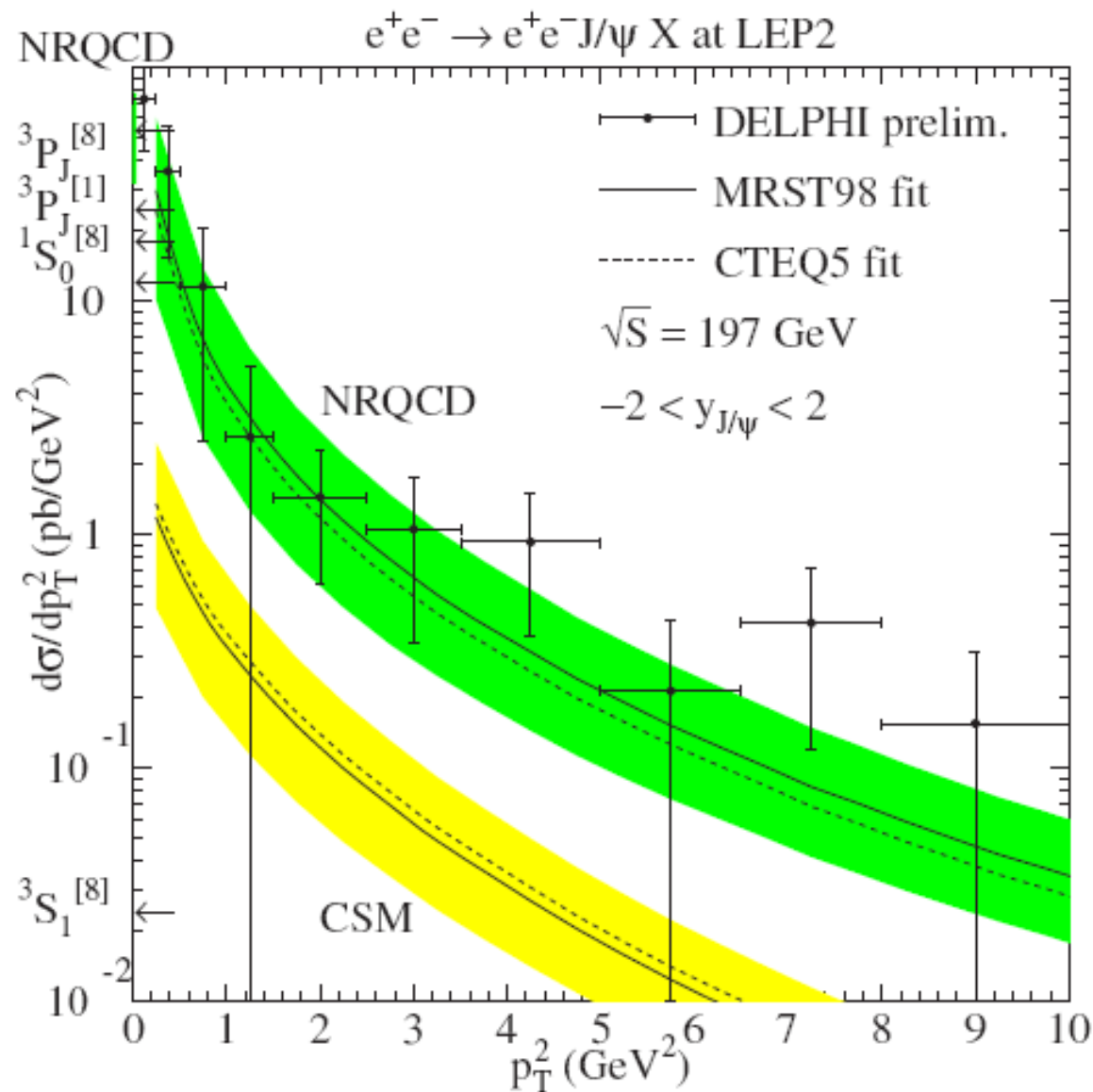
Wave function: $\Phi_{Q\bar{Q}}(r) \Rightarrow \Phi_{Q\bar{Q}}(r, T)$

J/ψ formation rate $\propto |\Phi_{Q\bar{Q}}(r, T)|^2$

J/ψ suppression \Leftrightarrow medium properties

- But, do we understand the production mechanism of J/ψ well enough to calibrate the production rate and to extract the information on QGP?

□ LEP data on J/ψ photo-production: $\gamma\gamma \rightarrow J/\psi + X$



QWG's report